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13. ABSTRACT (Maximum 200 words)

Understanding of the phenomenon of localization requires construction of micromechanically based constitutive equations for microcracking solids, involving a number of rather complex issues of the mechanics of multiple interacting defects. Therefore, the first part of our efforts was concentrated on several key issues of micromechanics: stress-induced anisotropy and the mechanics of multiple interacting microcracks in an anisotropic environment; mechanics of secondary ("winged") microcracks forming under compressive conditions; mechanics of intersecting cracks, both in two- and three-dimensional configurations; the impact of compressible fluid that saturates a geomaterial on the interaction effects. The second part of our effort was focused on localization of deformation and formation of shear bands in inhomogeneous brittle materials under compression. The conditions of localization and formation of shear bands are very sensitive to the exact structure of constitutive equations, in particular, to the factors like violation of the normality rule, formation of a vertex on a loading surface, dilatancy and stress-induced anisotropy. These issues have been examined.

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# **Final Technical Report to AFOSR**

**Project: Micromechanically based study of localization of deformation and shear bands formation in brittle materials**

**Grant # F49620-92-J-0429**

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Construction of micromechanically based constitutive equations for microcracking solids requires an understanding of a number of rather complex issues of the mechanics of multiple interacting defects. Therefore, our efforts were concentrated on the following problems of micromechanics.

Preferentially oriented microcracking under compressive conditions results in substantial stress-induced *anisotropy*. Therefore, it is important to understand the mechanics of multiple interacting microcracks in an anisotropic environment. (We note that these results, besides being relevant to the project, are also quite relevant for the mechanics of composite materials). A substantial effort was undertaken in this direction. The major findings, presented in papers 8, 9 and the dissertation of C.Mauge, can be described as follows.

- It was found that the impact of the material anisotropy on crack interactions is quite significant. The anisotropy of Young's modulus produces the strongest impact: it enhances (weakens) the interactions under a loading applied in the stiffest (softest) direction. Sensitivity to the shear modulus of the matrix is lower and the impact of Poisson's ratio is small.

- Effective moduli of a matrix with arbitrarily oriented cracks were found. An unexpected finding is that the approximation of non-interacting cracks (that constitutes the simplest approach to the problem) actually remains accurate at high crack densities. The underlying physical mechanism is that, although crack interactions are strong at high crack densities, the competing interaction effects of shielding and amplification cancel each other, provided the mutual positions of defects are random. This result was confirmed by extensive computer experiments on large sample arrays of strongly interacting cracks.

Since the secondary ("winged") microcracks forming under compressive conditions are curved, it is important to understand the impact of crack curvilinearity. We addressed this problem and analyzed both the stress intensity factors at the tips of curvilinear cracks and the full stress field generated by a curvilinear crack. The results are reported in the thesis of H. Matsczynska.

We investigated several physically important effects produced by microcrack interactions. We mention, in particular, the following problems (presented in detail in book chapter 2, section IV).

- The effects of stress amplification and stress shielding and their relative strength in two- and three-dimensional configurations. It was found that the effect of amplification (that enhances crack propagation and makes the effective moduli "softer" than in the approximation of non-interacting cracks) has relatively short range, whereas the opposite

effect of stress shielding has a much longer range. We also found that these effects are substantially stronger in the two-dimensional configurations, as compared to the three-dimensional ones.

-Range of influence of a crack in the environment of other cracks. The issue of "short range" and "long range" interactions provides an important insight into the mechanics of multiple interacting cracks, particularly in connection with localization of deformation. The problem was analyzed by perturbing the position and orientation of a representative crack in the otherwise periodic arrangement. We found that, with the exception of extremely dense crack arrays, the influence of a defect is, generally, limited to one or two closest neighbours only.

-Extremal properties of slightly asymmetric crack arrangements. We found that slight disturbances introduced into otherwise symmetric configurations result in an increase in the values of stress intensity factors. This fact may explain, for example, why "cracks avoid each other", when coalescing - the problem that received attention in the literature.

-Mechanics of intersecting cracks, both in two- and three-dimensional configurations. We found a simple way to extend the method of analysis of interacting cracks developed earlier to the configurations when cracks intersect each other. Comparison with several available test problems shows a good accuracy of our solutions. These solutions are of obvious relevance to the mechanics of dense crack arrays with frequent intersections.

Mechanics of narrow, crack-like cavities filled with compressible fluid (analyzed in detail in book chapter 2, section IV). This issue is of importance for the mechanics of damaged geomaterials that are fluid-saturated. We mention, in particular, the following developments.

-We found that the stress interactions between cavities are strongly coupled with the changes of fluid pressures in cavities, and established that this coupling is governed by one dimensionless constant - a product of the fluid compressibility, Young's modulus of the material and the aspect ratio of the cavity.

-We identified and analyzed the phenomenon of "polarization of fluid pressure" - dependence of the fluid pressure changes in cavities (when loading is applied) on the orientation of the cavity.

-We analyzed the impact of the fluid on the interaction effects. In particular, we found that the fluid "dampens" the interaction effects, whether the latter are the ones of shielding or the ones of amplification.

-We found that orthotropy of the effective elastic properties is lost when the crack cavities are fluid-filled. The underlying physical mechanism is that the fluid strongly affects the normal compliance of a crack. but leaves its shear compliance unchanged.

The second part of our effort was focused on localization of deformation and formation of shear bands in inhomogeneous brittle materials under compression. The previous studies of the subject focused on the phenomenological approach that does not explicitly reflect the development of micromechanical events. However, the conditions of localization and formation of shear bands are very sensitive to the exact structure of constitutive equations, in particular, to the factors like violation of the normality rule, formation of a vertex on a loading surface, dilatancy and stress-induced anisotropy. It appears, therefore, that a purely phenomenological analysis may not be adequate. We study the localization on the basis of constitutive equations that are micromechanically-based and reflect the actual inelastic mechanisms. In addition to making the analysis physically sound, such an approach allows one to link the macroscopic localization conditions to the micromechanical events.

This research is relevant for applications to a number of materials; among them: geomaterials, concrete and certain ceramics. In geophysics, for example, the investigations of localization shed light on the mechanics of large scale geological processes (including earthquakes, for which localization of deformation constitutes one of the main mechanisms); dependence of localization on various physical parameters (for example, on the porous pressure, for a fluid-saturated rock) is a problem of significant importance.

When a brittle elastic material is loaded by stresses that are compressive (but the principal stresses are different) then, the shear stresses, if they are sufficiently high, initiate local frictional slidings along interfacial microdefects (weak grain boundaries, microcracks, kerogen flakes in petroleum source rocks). Wedging action produced by these slidings drives the process of nucleation and propagation of "wing" tensile microcracks giving rise to a configuration of winged frictional sliding crack. The concept of winged crack was suggested in a number of papers but was later criticized by several authors on the grounds that such cracks have rarely been clearly identified in the SEM studies of geomaterials; rather, tensile microcracks seemed to originate at a variety of sources.

Micromechanics of local inelastic events may indeed vary and is by no means unique. However, several reasons can be given to support the relevance of the winged crack mechanism; among them the recent data suggesting that the focal mechanism for stress-induced acoustic emissions involves shear motion. Winged configurations were also directly observed in ice under compression and a micromechanism similar to a winged crack exists in semi-brittle ceramics. Note, also, that various sliding-driven

micromechanical events (for example, sliding grain that pushes sideways the neighbors) are, in fact, similar to the "winged" crack mechanism, in the sense that frictional sliding acts as a driving force for dilatant microstrains. We accept, therefore, the concept of a "winged" crack as at least one major micromechanism of inelasticity under compression in the elastic-brittle range of behavior.

The macroscopic stress-strain relations are obtained by averaging the microstrains produced by winged cracks (frictional slidings and opening of wing cracks being two sources of inelastic strains). These constitutive relations are path-dependent and have to be formulated in the incremental form (in this sense, they resemble the equations of metal plasticity). However, the presence of internal friction and dilatancy makes them distinctly different from the equations of plasticity.

Therefore, a considerable effort has been devoted to establishing sound constitutive equations that reflect the essential features of actual micromechanics of inelasticity - internal friction, dilatancy, stress-induced anisotropy. Thus, the constitutive equations are based on (a) sound micromechanical model of a single inelastic event - growth of a winged frictional sliding crack. Such a model should, in our opinion, be correct in the asymptotics of short wings (when the crack just starts to grow) and, particularly, in the asymptotics of long wings (the latter make the dominant contribution to the overall strain); (b) incorporation of strong interaction of such winged cracks into the model. This is essential for the stages of well developed microcracking and, in particular, for the localization problem. It will be done on the basis of the method of analysis of many cracks' problems developed earlier by the PL. However, since this method in its original form has been formulated for rectilinear traction free cracks, it required a further development, to account for the fact that interacting objects - winged cracks - are substantially different from rectilinear traction-free cracks. Formulation of constitutive equations is then incorporated into the general framework of thermodynamics with internal variable, using the formalism developed by Rice; the latter appears to be the most convenient tool for transition from the micro- to the macrolevel of description.

We also investigate the effect of presence of fluid in crack cavities (fluid-saturated materials). This factor may alter the overall behavior of the material quite significantly. In particular, the fluid tends to dampen the interaction effects.

We undertook an in-depth study of the localization process along the above mentioned lines. More specifically, some of the findings are as follows:

- At the stage of frictional sliding on pre-existing cracks, localization does not occur, even at very high ratios of the maximal compressive stress to the lateral stress.

- At the stage of small wings, localization does not occur, either (although small wings tend to promote the tendency to localization).

- At the stage of long wings, we have a clear-cut localization. Our analysis gives the values of applied stresses at which the localization occurs, in terms of the micromechanical parameters (friction coefficient, fracture toughness of the material, density of the initial sliding microcracks).

As expected, lowering of the friction coefficient or increase in the initial microcrack density facilitates localization. In addition, we found that the increase in the average size of the microcracks while keeping their density the same (smaller number of larger cracks) facilitates localization as well.

Some of these results are illustrated in Fig.1.

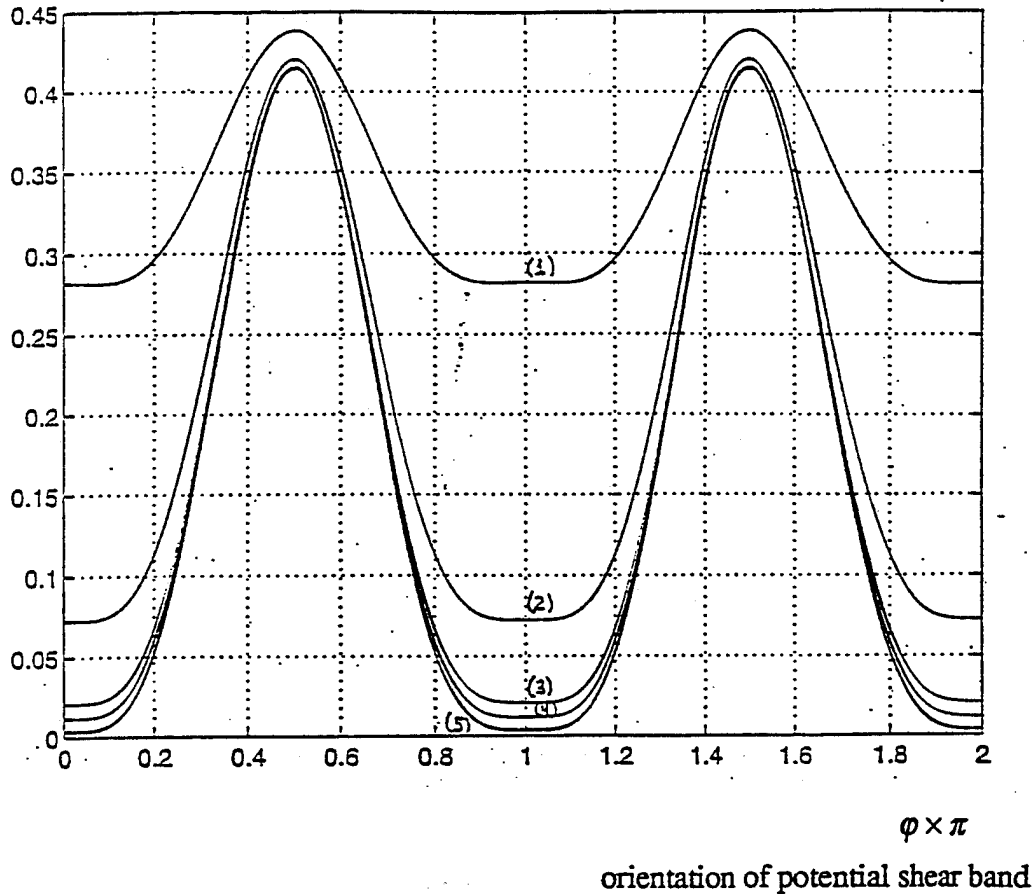


Fig. 1. Tendency to localization and shear band formation at the stage of well developed microcracking. ( $\lambda = \sigma_2 / \sigma_1$  - stress ratio; friction coefficient = 0.6; crack density = 0.2)

$$\lambda^{(1)} = 10/300, \lambda^{(2)} = 10/500, \lambda^{(3)} = 10/800, \lambda^{(4)} = 10/1000, \lambda^{(5)} = 10/1700$$

## Publications

### ◇ BOOKS AND BOOK CHAPTERS:

1. Micromechanics of Brittle Solids with Defects, Cambridge U. Press (work underway)
2. Elastic Solids with Many Cracks and Related Problems, in *Advances in Applied Mechanics*, ed. by J.Hutchinson and T.Wu, Academic Press, pp. 259-445 (1994).

### ◇ PAPERS:

1. On Continuum Characterization of Crack Arrays and Its Limitations, in Recent Advances in Damage Mechanics and Plasticity (ed. F. Ju), ASME, pp. 103-115, 1992.
2. "Effective Elastic Properties of Cracked Solids: Critical Review of Some Basic Concepts", Applied Mechanics Reviews, v. 45, no.8, pp. 305-336, 1992.
3. "Interacting Arbitrarily Oriented Cracks in Anisotropic Material. Stress Intensity Factors and Effective Properties", with C.Mauge, International Journal of Fracture, v.58, pp.R69-74, 1992.
4. "On the Effective Moduli of Solids with Cavities and Cracks", International Journal of Fracture, v.59, pp.R17-21, 1993.
5. "On the Effective moduli of Cracked Solids", in Proc. of 11-th Symp. on Energy Sciences, pp.70-76, 1993.
6. "Solids with Holes of Irregular Shapes: Effective Moduli and Anisotropy", with I.Tsukrov, International Journal of Fracture, v. 64, pp. R9-12, 1993.
7. "Effective Properties of Solids with Cavities of Various Shapes", with I.Tsukrov and B.Shafiro, Applied Mechanics Reviews, v. 47, no.1 pp. 151-174, 1994.



8. "Effective Elastic Properties of Anisotropic Materials with Arbitrarily Oriented Cracks", with C.Mauge, Journal of the Mechanics and Physics of Solids, v.42, pp. 1-24, 1994.
9. "Stress Intensity Factors of Interacting Cracks Arbitrarily Oriented in Anisotropic Matrix", with C.Mauge, International Journal of Fracture, v.65, pp.115-139, 1994.
10. "Effective Properties of Solids with Randomly Located Defects", with I.Tsukrov and B.Shafiro, in Probabilities and Materials, ed. by D.Breusse, pp. 225-240, 1994.
11. "Stress Concentrations and Microfracturing Patterns for Interacting Elliptical Holes", with I.Tsukrov, International Journal of Fracture, v. 68, 89-92, 1994.
12. "On Anisotropy of Solids with Non-Randomly Oriented Cavities", with I.Tsukrov and B.Shafiro, in Fracture and Damage in Quasibrittle Structures, ed. by Z.Bazant et al, Chapman and Hall, pp. 19-24, 1994.
13. "On the Concept of Damage under Creep Conditions and in Brittle-Elastic Materials", International Journal of Damage Mechanics, v. 3, pp. 329-337, 1994.
14. "Longwave Speeds in Materials with Cracks and Cavities of Various Shapes", with B.Shafiro, in Review of Progress in QNDE, Proc. of 21-st Annual Meeting on Non-Destructive Evaluation, July, 1994, Colorado.
15. "Microcrack-induced Elastic Wave Anisotropy of Brittle Rock", with C.Sayers, Journal of Geophysical Research, v.100 (B3), pp. 4149-4156, 1995.
16. "On Stress-Strain Relations for Cracked Elastic Materials in Compression", with F.Lehner, in Mechanics of Jointed and Faulted rocks, ed. by Rossmanith, pp.49-61, 1995.
17. "On the Effective Elastic Properties of Solids with Cavities of Various Shapes", in Mechanics of Jointed and Faulted rocks, ed. by Rossmanith, pp.81-84, 1995.
18. "Three-Dimensional Interactions of a Half-Plane Crack with Point Forces, Dipoles and Moments", with E.Karapetian, International Journal of Solids and Structures, in press.

### **Graduate Students supported, fully or partially, by the grant**

Christophe Mauge, Ph.D. defended in 1993

Abraham Shurland, MS defended in 1993

Igor Tsukrov, MS defended in 1993, Ph.D. defended in 1995

Hanna Matsczynska, MS defended in 1993

Boris Shafiro, MS defended in 1994; Ph.D. defended in 1996

### **Conference presentations**

International Symposium on Ice, Banff, Canada (two lectures) 1992.

ASME Annual Meeting, Tucson, Arizona, 1992.

International Congress of Theoretical and Applied Mechanics, Haifa, Israel, 1992.

Symposium on Basic Energy Sciences, Argonne National Laboratory, 1993.

ASME-ASCE-SES Meeting, Charlottesville, VA, 1993.

Workshop "Probabilities and Materials" (invited lecture), France, 1993.

Annual Meeting "Review of Progress in Quantitative NDE", Colorado, 1994.

US-Europe Workshop "Fracture and Damage in Brittle Structures", Prague, 1994.

International Conference on Geomechanics, Vienna, Austria (invited lecture), 1995.

Symposium "Continuum Mechanics and Discrete Models", Varna, Bulgaria, 1995.

"Numiform-95", International Conference on Numerical Methods, Cornell U, 1995

Workshop "Mechanical Response of Damaged Solids", Fontainebleau, France, 1995  
(invited principal lecturer, seven hours of lectures given)

International Conference on Continuous and Discrete Systems, Varna, Bulgaria, 1995.

### **Seminars and invited lectures**

Harvard U. (1993); Brown U. (1995); Swiss Federal Institute of Technology (1992); ALCOA Tech. Center (1992); U. of Innsbruck, Austria (1992, 1994); U. of California, Berkeley (1992); Cornell U. (1992), Nuclear Research Center, Israel (1992); U. of Minnesota (1993), U. of Newfoundland (1993, 1994); NIST (1993); Schlumberger Research, England (1993); Centre for Materials at Ecole des Mines, Paris, France (1993); Technical U. of Vienna (1993); U. of Colorado (1994); Cold Regions Eng-g Research Laboratory, N.H.(1994); MIT (1995); Czech Academy of Sciences (1995).